Transmission and Reflection of Sound by a Blade Row

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Theme

THE interaction in an axial-flow compressor or fan of a rotor with the flowfield produced by an adjacent stator (or vice versa) produces sound waves of discrete frequency. In many cases these waves must propagate through adjacent blade rows before escaping from the fan. This propagation problem, which is of interest in aircraft engine noise abatement programs, is the subject of the present paper. The two-dimensional transmission and reflection of a plane sound wave impinging on a lattice of flat plate airfoils in a subsonic stream is treated in the limit of wavelength long compared to the plate spacing and chord. Linearized flow is assumed, permitting the superposition of a small finite steady loading without affecting the final acoustic results. A quasi-steady Prandtl-Glauert flow analysis near the airfoils combined with a far-field acoustic solution leads to simple closed-form solutions for the transmitted and reflected wave amplitudes. Comparison with the arbitrary wavelength, numerical solution of Kaji and Okazaki shows good agreement for the longer wavelengths.

Content

The hub/tip ratio of the fan is assumed to be near unity, permitting the annular region of the fan to be approximated by the two-dimensional geometry shown in Fig. 1. The wavelength is assumed long compared to the spacing d and chord c of the flat plate airfoils. The case of the incident wave moving upstream onto the lattice is shown, but the results are given for both this case and the case of the incident wave moving downstream. For this latter case the same figure applies except that the direction of the Mach number is reversed. The analysis assumes small perturbations of a uniform, inviscid, isentropic flow of Mach number, M. The governing equation is thus the familar linearized equation

$$\frac{1}{a_o^2}\frac{\partial^2 P}{\partial t^2} - \nabla^2 P + \frac{2M}{a_o}\frac{\partial^2 P}{\partial x \partial t} + M^2\frac{\partial^2 P}{\partial x^2} = 0 \qquad (1)$$

where a_o is the speed of sound, P is the perturbation pressure, x is distance (Fig. 1), and t is time. The Kutta condition is assumed to be always satisfied at the trailing edges of the blades.

The long wavelength assumption allows the flowfield to be divided into near and far fields. The near field is treated as a quasi-steady Prandtl-Glauert flow, and incompressible, steady lattice theory[1] is used to relate the blade lift force to the fluctuating angle-of-attack α (Fig. 1) produced by the sound wave. The far-field problem is simplified by the fact that only the total lift force is important, rather than its distribution on the blade. This solution relates the unknown lift force and α to the incident wave amplitude. Since the acoustic wavelength is assumed to be much greater than the spacing and chord of the lattice, the near and far-field results have a region of overlap corresponding to the geometric far field but acoustic near field. Equating the α given by the far-field expression to that of the near-field expression determines the blade loading from which the transmitted pressure P_t and the reflected pressure P_r relative to the incident pressure P_i can be calculated. The results are

$$P_t/P_i = 1 - M\sin^2\alpha/(M\Gamma_{u,d} + \mathcal{L}\psi_\pm)$$
$$P_r/P_i = M\sin\alpha(2\psi_\pm\sin\theta - \Theta^2\sin\alpha)/\Theta^2(M\Gamma_{u,d} + \mathcal{L}\psi_\pm) \qquad (2)$$

where

$$\psi_\pm \equiv \cos(\theta - \alpha) \pm M\cos\theta$$
$$\Gamma_u \equiv \sin^2\alpha \qquad (3)$$
$$\Gamma_d \equiv (2\psi_+\sin\theta - \Theta^2\sin\alpha)^2/\Theta^2(\Theta^2 + 2M\psi_+\cos\theta)$$

and θ, d, and c (and the Mach number) are contained in the parameter $\mathcal{L}$ defined by

$$\mathcal{L} \equiv \frac{R}{2\nu}\frac{\beta}{\Theta} + \frac{\beta^2}{\Theta^2}\cos\theta \qquad (4)$$

where

$$\beta^2 \equiv 1 - M^2 \qquad \Theta^2 \equiv 1 - M^2\cos^2\theta \qquad (5)$$

and R and ν are parameters used in incompressible lattice theory[1] and are given by the simultaneous equations

$$R = (\nu^4 + 2\nu^2\cos 2\theta' + 1)^{1/2}$$
$$\frac{\pi c'}{2d'} = \cos\theta' \ln\left(\frac{R + 2\nu\cos\theta'}{1 - \nu^2}\right) + \sin\theta' \tan^{-1}\left(\frac{2\nu\sin\theta'}{R}\right) \qquad (6)$$

The primed variables c', d', and θ' refer to the dimensions of a Prandtl-Glauert transformed lattice so that

$$c' = c/\beta \qquad d' = d\cos\theta/\cos\theta' = d\Theta/\beta$$
$$\theta' = \tan^{-1}[(1/\beta)\tan\theta] \qquad (7)$$

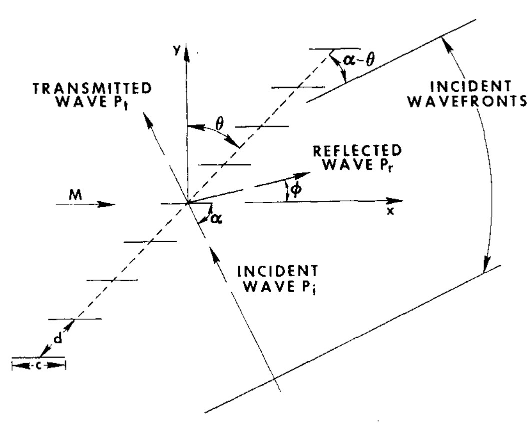

Fig. 1 Sound wave incident on a lattice of flat plate airfoils.

Presented as Paper 71-181 at the AIAA 9th Aerospace Sciences Meeting, New York, January 25–27, 1971; submitted February 24, 1971; synoptic received June 28, 1971. Full paper is available from AIAA. Price: AIAA members, \$1.50; nonmembers, \$2.00. Order must be accompanied by remittance.

Index categories: Aerodynamic and Powerplant Noise (Including Sonic Boom); Aircraft Propulsion System Noise.

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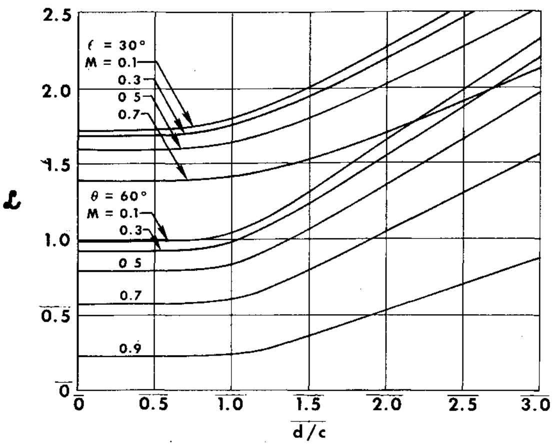

Fig. 2 Lattice parameter ℒ.

Equation (2) includes the cases of both the upstream propagating incident wave and the downstream propagating incident wave. For the former Γ_u and ψ_- are used, whereas for the latter Γ_d and ψ_+ are applicable.

The lattice parameter ℒ is plotted in Fig. 2 as a function of d/c for various values of M and θ. It can be seen that ℒ quickly approaches a constant value dependent on M and θ when d/c becomes less than 1. Also, ℒ approaches a linear function of d/c when $d/c \gg 1$. The expressions for these limiting cases are

$$\lim_{d/c \to 0} \mathcal{L} = (2\beta^2/\Theta^2)\cos\theta$$

$$\lim_{d/c \to \infty} \mathcal{L} = \frac{2}{\pi}\beta\frac{d}{c} + \frac{\beta^2}{\Theta^2}\cos\theta \tag{8}$$

For these two cases Eqs. (4, 6, and 7) can be replaced with the much simplified expressions given in Eq. (8).

Results for the case of an upstream propagating incident wave are shown in Fig. 3 plotted vs the incidence angle $\theta - \alpha$ defined in Fig. 1. Kaji and Okazaki's numerical results are shown as dashed lines for comparison. Here $\kappa = \omega/a_o$, where ω is the circular frequency measured in blade-fixed coordinates. Since the transmitted wave is fairly insensitive to wavelength λ, good agreement results for the range of wavelengths considered. For the reflected wave, good agreement is obtained only for the longer wavelengths. The relation between κ and λ is

$$\kappa/2\pi = (1 - M\cos\alpha)/\lambda \tag{9}$$

Thus, a value for κ of 0.7 π/c corresponds to $\lambda/c \approx 2$.

Further results are plotted in the backup paper[4] where it is shown that increasing M decreases the transmitted sound.

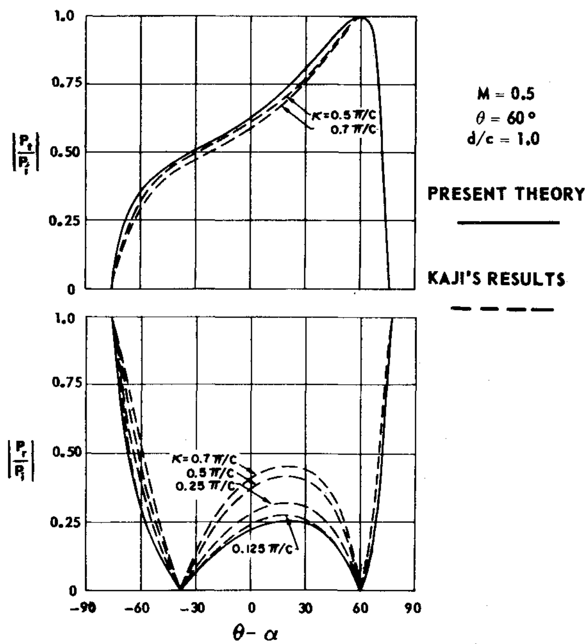

Fig. 3 Variation of transmitted and reflected pressures with incidence parameter θ-α for the case of upstream propagating incident waves.

When $M \to 1$, Eq. (4) shows that $\mathcal{L} \to 0$, and hence $P_t \to 0$ for the case of an upstream propagating incident wave.

Kaji and Okazaki did not present results based on their thin airfoil theory for the case of a downstream propagating incident wave. However, excellent agreement is obtained when comparison is made with the predictions of their semiactuator disk theory[2] for this case. When the wavelength is made shorter than a few blade spacings, higher order modes of the lattice are excited and the present theory can no longer be applied. In this case either the Weiner-Hopf theory of Mani and Horvay[3] or the airfoil theory of Kaji and Okazaki must be used. Where it is applicable, however, the present theory provides a great simplification over either of these two analyses.

References

[1] Durand, W. F., ed., "Aerodynamic Theory," Vol. 2, Dover, New York, 1963, pp. 91–96.

[2] Kaji, S. and Okazaki, T., "Propagation of Sound Waves Through a Blade Row," *Journal of Sound and Vibration*, Vol. 11, No. 3, March 1970, pp. 339–375.

[3] Mani, R. and Horvay, G., "Sound Transmission Through Blade Rows," *Journal of Sound and Vibration*, Vol. 12, 1970, pp. 59–83.

[4] Amiet, R. K., "Transmission and Reflection of Sound by a Blade Row," AIAA Paper 71-181, New York, 1971.